

## QUAKER VALLEY SCHOOL DISTRICT

### 3412 Pre-Calculus and Discrete Math

#### COURSE DESCRIPTION

Pre-calculus topics include a review of the elementary functions, advanced properties of functions, (including special attention to polynomial and rational functions), polar coordinates, complex numbers, and introductions to the derivative and integral. Discrete mathematics topics include recursion, induction, combinatorics, vectors, graphs, and circuits. Manipulation of complex rational expressions, not emphasized in previous courses, is discussed here. Mathematical thinking, including specific attention to formal logic and proof and comparing structures, is a unifying theme employed throughout the course.

Note: The letters following the units of instruction correspond to the International Baccalaureate areas of interaction. They are as follows: AL – Approaches to Learning, CS – Community Service, E – Environment, HF – Homo Faber, HS – Health & Social Education.

#### PENNSYLVANIA ACADEMIC STANDARDS

##### Reading, Writing, Listening and Speaking

- 1.1 Learning to Read Independently
  - 1.2 Reading Critically in All content Areas
  - 1.3 Reading, Analyzing and Interpreting Literature
  - 1.4 Types of Writing
  - 1.4 Quality of Writing
  - 1.6 Speaking and Listening
  - 1.7 Characteristics and Function of the English Language
  - 1.8 Research
- Mathematics

- 2.1 Numbers, Number Systems and Number Relationships

- 2.2 Computation and Estimation
- 2.3 Measurement and Estimation
- 2.4 Mathematical Reasoning and Connections
- 2.5 Mathematical Problem Solving and Communication
- 2.6 Statistics and Data Analysis
- 2.7 Probability and Predictions
- 2.8 Algebra and Functions
- 2.9 Geometry
- 2.10 Trigonometry
- 2.11 Concepts of Calculus

#### COURSE TEXT AND SUPPLEMENTAL MATERIALS

Pre-calculus and Discrete Mathematics 2nd edition, UCSMP, Scott Foresman, 1998

#### MAJOR UNITS AND ACTIVITIES

The chapters will be covered in the order that they are presented in the textbook. However, some chapters fall under unit headings that are not in the same chronological order. Assignments will consist of reading the textbook and completing the questions provided at the end of each section. Additional practice on material presented in the book, extended projects, and writing assignments may also be assigned.

Logic and Graph Theory; Chapters 1, 11

Logic

Skills

- Identify forms of logic statements
- \* Write logically equivalent forms of statements.

- \* Write the negation of a logic statement.
- \* Determine the truth-value of a statement.

### Properties

- \* Identifying properties of logical statements
- \* Use substitutions to verify specific statements
- \* Determine whether arguments are valid or invalid
- \* Use logic to prove or disprove statements

### Uses

- \* Determine the truth of quantified statements outside of mathematics
- Determine whether or not a logical argument outside of mathematics is valid

### Representations

- Write truth tables for logical expressions
- Translate logic networks into logical expressions and input-output tables and determine output signals

### Graphs and Circuits

#### Skills

- Draw graphs given sufficient information

#### Properties

- Identify parts of graphs and types of graphs
- Determine whether there exists a graph containing vertices with given degree
- Determine whether a graph has an Euler Circuit.

#### Uses

- Use graphs to solve scheduling and probability problems
- \* Use the total degree of a graph theorem and its corollaries to solve handshake problems.
- \* Solve application problems involving circuits.

#### Representations

- Convert between the picture of a graph or directed graph and its adjacency matrix

\* Use the powers of the adjacency matrix of a graph to find the number of walks of a given length from a given starting vertex to a given ending vertex

## Functions, Equations and Inequalities; Chapters 2, 3, 6

### Analyzing Functions

#### Skills

· Determine relative minima and maxima of a function and intervals on which it is increasing or decreasing

\* Re-write exponential and logarithmic expressions and equations

#### Properties

· Identify the domain, range and minimum and maximum values of functions

\* Determine the end behavior of a function

#### Uses

\* Use trigonometric, exponential and logarithmic functions as models

\* Solve maxima-minima problems

#### Representation

· Analyze a function from its graph

## Functions, Equations and Inequalities

### Skills

· Solve equations by applying a function to each side, taking into account non-reversible steps

\* Describe the sum, difference, product quotient and composite of two given functions

\* Find zeros of functions and solutions to equations and factoring or chunking

\* Use the intermediate value theorem and the bisection method to locate or approximate zeros of a function

\* Solve inequalities algebraically

## Properties

- \* Analyze the reversibility of steps used in solving equations and inequalities
- Identify and prove properties of inverse functions
- \* Identify continuous functions and their properties
- \* Identify the amplitude, period and phase shifts of trigonometric function

## Uses

- \* Apply equation-solving techniques to real-world problems
- \* Use operations on functions to find formulas which model realistic situations
- \* Use inequalities to solve real-world problems

## Representations

- \* Graph functions obtained from other functions by function operations or inverses
- \* Find an equation of a graph after a transformation use graphs to approximate zeros of functions and solve equations and inequalities

## Trigonometric Identities and Equations

### Skills

- \* Without a calculator, use trigonometric identities to express values of trigonometric functions in terms of rational numbers and radicals.
- \* Evaluate inverse trigonometric functions with or without a calculator.
- \* Solve trigonometric equations and inequalities algebraically

### Properties

- Prove trigonometric identities and identify their domains

### Uses

- \* Solve problems using inverse trigonometric functions.
- \* Use trigonometric equations and inequalities to solve applied problems

### Representations

- \* Use an automatic grapher to test proposed trigonometric identities
- \* Use graphs to solve trigonometric equations and inequalities

Numbers and Functions; Chapters 4, 5, 8, 12

Integers and Polynomials

Skills

- \* Use the Quotient-Remainder Theorem for integers or polynomials.
- \* Use synthetic substitution to find values of polynomial functions
- \* Divide polynomials
- \* Determine the congruence of integers for a given module
- \* Factor polynomials

Properties

- Justify properties of factors of integers or factors of polynomials
  - \* Use the properties of congruence of integers for a given module to rewrite sentences
  - \* Use the remainder theorem, factor theorem or quotient remainder theorem to describe characteristics of given polynomials
  - \* Use proof by contradiction
  - \* Use the factor search theorem and the fundamental theorem of arithmetic in determining prime numbers and prime factorizations

Uses

- Use modular arithmetic to solve applied problems

Representations

- Represent numbers in other bases and perform addition in Base 2

Rational Numbers and Rational Functions

Skills

- \* Find values of trigonometric functions
- \* Identify numbers as rational or irrational
- \* Simplify expressions involving radicals
- Simplify rational expressions
- Solve rational equations

Properties

- \* Prove properties of rational and irrational numbers
- \* Use limit notation to describe the behavior of rational functions
- \* Classify discontinuities as essential or removable

#### Uses

- Apply rational expressions and rational equations

#### Representations

- Graph quotients of polynomials of trigonometric functions
- Relate the limit of a function to its graph and find equations for its asymptotes
- Use right triangles to find values of trigonometric functions

### Polar Coordinates and Complex Numbers

#### Skills

- \* Express complex numbers in a  $+bi$ , rectangular, polar and trigonometric form
- \* Perform operations with complex numbers convert between polar and rectangular coordinate representations of points
- \* Find powers and roots of complex numbers
- \* Find all zeros and their multiplicities of a given polynomial

#### Properties

- \* Prove or verify properties of complex numbers
- \* Use the properties of polynomials to find or describe their zeros

#### Uses

- Use complex numbers to solve AC circuit problems

#### Representations

- Graph complex numbers and verify the Geometric Addition and Geometric multiplication theorems

- \* Sketch graphs of polar equations

## Vectors

### Skills

- \* Find the magnitude and direction of two-dimensional vectors
- \* Find sums, opposites, scalar products and dot products of two-dimensional vectors
- \* Find sums, lengths, scalar products, dot products and cross products of vectors in 3-space  
find the measure of the angle between two vectors

### Properties

- \* Prove or disprove statements about vector operations
- Identify parallel and orthogonal vectors

### Uses

- \* Use vectors in a plane to decompose motion or force into x- and y- components
- \* Use addition of vectors in a plane to solve problems involving forces or velocities

### Representations

- \* Represents two-dimensional vectors in their component or polar representation or as directed segments
- \* Represent addition, subtraction and scalar multiplication of two-dimensional vectors graphically
- \* Geometrically interpret three-dimensional vectors and their operations
- \* Represent lines in a plane using vector or parametric equations
- \* Represent lines, planes and spheres in 3-space using parametric, vector or coordinate equations

## Recursion and Counting; Chapters 7, 10

## Recursion and Mathematical Induction

### Skills

- \* Determine terms of a sequence defined either explicitly or recursively
- \* Conjecture explicit formulas for recursively defined sequences
- \* Use summation notation to write sums

- \* Rewrite sums recursively evaluate a finite or infinite geometric series

### Properties

- \* Prove that a recursively-defined sequence has a particular explicit formula
- \* Prove statements using mathematical induction

### Uses

- \* Use recursive thinking to solve problems
- Execute algorithms on sets of numbers

### Representations

- Interpret computer programs which calculate terms of sequences

### Combinatorics

#### Skills

- Describe the essential features of counting problems
- Evaluate expressions indicating permutations or combinations
- Apply the binomial theorem to expand binomials or find specific terms

#### Properties

- \* Use properties of permutation and combinations to prove identities
- \* Apply the binomial theorem to deduce properties of sets

#### Uses

- Use the Multiplication Counting Principle and permutations to solve counting problems
- \* Use combinations and the Binomial Theorem to solve counting problems
- Find binomial probabilities in realistic situations

#### Representations

- Use a possibility tree to determine the number of outcomes in a given situation

## Calculus; Chapter 9

### The Derivative in Calculus

#### Skills

- \* Compute average rates of change in functions
- \* Use the definition of derivatives to compute derivatives

#### Properties

- Use derivatives to identify properties of functions

#### Uses

- \* Find rates of change in real situations
- \* Use derivatives to find velocity and acceleration of a moving object
- \* Use derivatives to solve optimization problems

#### Representations

- Relate average rate of change to secant lines of graph functions
- \* Determine properties of derivatives from the graph of a function

Standards for all: 2.1.11A; 2.2.11A; 2.2.11B; 2.2.11D; 2.2.11E; 2.2.11F; 2.4.11A; 2.4.11B; 2.4.11E; 2.5.11A; 2.5.11B; 2.5.11C; 2.5.11D; 2.8.11N; 2.8.11O; 2.8.11Q; 2.8.11R; 2.8.11S; 2.8.11T; 2.9.11G; 1.1.11A; 1.1.11C; 1.1.11D; 1.1.11E; 1.1.11F; 1.1.11H; 1.2.11A; 1.2.11B; 1.4.11B; 1.4.11D; 1.5.11A; 1.5.11B; 1.5.11C; 1.5.11F; 1.5.11G; 1.6.11A; 1.6.11C; 1.6.11D; 1.6.11E; 1.6.11F; 1.8.11A; 1.8.11B; 1.8.11C

#### TEACHING METHODS

The instructor will employ a variety of teaching methods that will include:

- Cooperative Learning
- Demonstration
- Experiential/Inquiry Learning
- Guided Practice

- Lecture/Discussion
- Role Play
- Simulation

#### RESEARCH COMPONENT

- Each chapter in the textbook provides a selection of research projects.
- These projects are suitable for individual work and small group work.
- \* Projects will be selected as appropriate for each chapter in the text.

#### ASSESSMENT TOOLS

The instructor will employ a variety of assessment methods that will include:

- Daily Homework
- Essays
- Experiments
- IB Assessments
- Journals/Notebooks
- Oral Presentation
- Reading Response Journals
- Student Projects
- Tests/Quizzes

#### TECHNOLOGY

- Students will use a graphing calculator throughout most of the course.

Planned courses for exceptional students shall be modified as needed per the student's Individual Education Plan.